Dynamics of Supersymmetric $SU(n_c)$ and $USp(2n_c)$ Gauge Theories

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ABSTRACT: We study dynamical flavor symmetry breaking in the context of a class of $N = 1$ supersymmetric $SU(n_c)$ and $USp(2n_c)$ gauge theories, constructed from the exactly solvable $N = 2$ theories by perturbing them with small adjoint and generic bare hypermultiplet (quark) masses. We find that the flavor $U(n_f)$ symmetry in $SU(n_c)$ theories is dynamically broken to $U(r) \times U(n_f - r)$ groups for $n_f \le n_c$. In the $r = 1$ case the dynamical symmetry breaking is caused by the condensation of monopoles in the $\underline{n_f}$ representation. For general r, however, the monopoles in the $\underline{{}_{n_f}C_r}$ representation, whose condensation could explain the flavor symmetry breaking but would produce too-many Nambu–Goldstone multiplets, actually "break up" into "magnetic quarks" which condense and induce confinement and the symmetry breaking. In $USp(2n_c)$ theories with $n_f \le n_c+1$, the flavor $SO(2n_f)$ symmetry is dynamically broken to $U(n_f)$, but with no description in terms of a weakly coupled local field theory. In both $SU(n_c)$ and $USp(2n_c)$ theories, with larger numbers of quark flavors, besides the vacua with these properties, there exist also vacua with no flavor symmetry breaking.

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1. An interesting phenomenon has been observed in $N = 2$ supersymmetric $SU(2)$ gauge theories with various flavors and with adjoint mass perturbation [1, 2]: confinement is caused by condensation of magnetic monopoles carrying nontrivial flavor quantum numbers (see also [3] for further details): spontaneous flavor symmetry breaking is caused by the same dyamical mechanism responsible for confinement in these models. We wish to know what happens in more general classes of models, and through a systematic analysis, to gain a more microscopic understanding of these phenomena and related ones in Quantum Chromodynamics. As we see below, the generalization from $SU(2)$ to higher-rank gauge groups turns out to be quite subtle.

We discuss here models constructed from exactly solvable $N = 2$ $SU(n_c)$ and $USp(2n_c)$ gauge theories with all possible numbers of flavor compatible with asymptotic freedom, by perturbing them with a small adjoint mass (reducing supersymmetry to $N = 1$) and keeping small, generic bare hypermultiplet (quark) masses. The advantage of doing so is that the only vacua retained are those in which the gauge coupling constant grows in the infrared. Another advantage is that in this way all flat directions are eliminated and one is left with a finite number of isolated vacua; keeping track of this number allows us to perform highly nontrivial checks of our analyses at various steps. Our analysis heavily relies on the breakthrough works by Seiberg and Witten [1, 2], and those which followed them [4]. Also crucial will be Seiberg's $N = 1$ electromagnetic duality [5, 6], and newly discovered universal classes of (super) conformally invariant theories [5]-[8].

The special cases of $SU(2) = USp(2)$ theories with $n_f = 1, 2, 3, 4$ were studied in [2]. For $n_f = 1, 4$, there is no dynamical flavor symmetry breaking. For $n_f = 2$, monopoles in the $(\underline{2}, 1)+(1, \underline{2})$ (spinor) representation of the flavor $[SU(2) \times SU(2)]/Z_2 = SO(4)$ group is found to condense after $N = 1$ perturbation $\mu \, \mathrm{Tr}\, \Phi^2$: the flavor symmetry is necessarily broken to $U(2)$. For $n_f = 3$, monopoles in the $\underline{4}$ (spinor) representation of the flavor $SO(6)$ group condense with $\mu \neq 0$ and the flavor symmetry is broken to $U(3)$ while there is another vacuum where a flavor-singlet dyon condenses and the flavor symmetry is unbroken. This result naturally leads to a conjecture that the condensation of monoples with non-trivial flavor transformation property explains the confinement *à la* 't Hooft [9] and the flavor symmetry breaking simultaneously. However, a simple thought reveals a problem with this picture. As we will see later, the monopoles in $USp(2n_c)$ theories transform under the spinor representation of $SO(2n_f)$ flavor symmetry, and their effective low-energy Lagrangian coupled to the magnetic $U(1)$ gauge group would have an accidental $SU(2^{n_f-1})$ flavor symmetry, and their condensation would lead to far too many Nambu–Goldstone multiplets. The case of $SU(2)$ gauge theories was special because the flavor symmetries of the monopole action precisely coincide with the symmetry of the microscopic theories due to the small number of flavors. This argument suggests that the phenomenon of flavor symmetry breaking is richer in higher rank theories.

Argyres, Plesser and Seiberg [10] studied higher-rank $SU(n_c)$ theories with $n_f \leq 2n_c - 1$ (asymptotically free) in detail. They showed how the non-renormalization theorem of the hyperKähler metric on the Higgs branch could be used to show the persistence of unbroken non-abelian gauge group at the "roots" of the Higgs branches (non-baryonic and baryonic branches) where they intersect the Coulomb branch. Some isolated points on the non-baryonic roots with $SU(r)$ $(r \leq [n_f/2])$ gauge

group as well as the baryonic root (single point) with $SU(\tilde{n}_c) = SU(n_f - n_c)$ gauge group were found to survive the $\mu \neq 0$ perturbation. Their main focus, however, was the attempt to "derive" Seiberg's duality between $SU(n_c)$ and $SU(\tilde{n}_c)$ gauge theories relying on the baryonic root,[1] and the issue of flavor symmetry breaking was not studied at any depth. The analysis also left a puzzle why there were "extra" theories at the non-baryonic roots which seemingly had nothing to do with Seiberg's dual theories. Another paper by Argyres, Plesser and Shapere addressed similar questions in $SO(n_c)$ and $USp(2n_c)$ theories [11].

In the present paper, we find that the flavor $U(n_f)$ symmetry in $SU(n_c)$ theories can be dynamically broken to various $U(r) \times U(n_f - r)$ groups. We find that in the $r = 1$ vacua the dynamical symmetry breaking is indeed caused by the condensation of monopoles in the $\underline{n_f}$ representation. For general r, however, the monopoles in the $\underline{{}_{n_f}C_r}$ representation, whose condensation could have explained the flavor symmetry breaking but would have produced too-many Nambu–Goldstone multiplets, actually "break up" into "magnetic quarks" whose baryonic composites under the unbroken $SU(r)$ gauge group match the monopoles. The baryonic roots are shown always to coincide with the non-baryonic roots with $r = \tilde{n}_c$. The non-baryonic roots are shown to be necessary ingredients of the Seiberg's dual theories rather than being "extra." The vacua with unbroken flavor symmetries are associated with the baryonic roots. The situation with $USp(2n_c)$ theories is even less trivial. The low-energy theories are non-trivial superconformal theories with no description in terms of a weakly coupled local field theory. In obtaining these results, counting of the number of vacua proved to be an extremely useful tool. The counting was done in the semi-classical limit, large μ limit, using the curve, as well as using low-energy effective Lagrangians and they all agree with each other.

2. First we perform a preparatory analysis, by minimizing the scalar potential following from the Lagrangian valid in the semi-classical regime (when both μ and m are large). $N = 1$ supersymmetry and holomorphy guarantee the absence of phase transitions between large μ, m to small μ, m. Therefore these vacua are related to quantum vacua in other regimes one by one.

The Lagrangian of the models has the structure

$$\mathcal{L} = \frac{1}{8\pi} \mathrm{Im}\, \tau_{cl} \left[\int d^4\theta \, \Phi^\dagger e^V \Phi + \int d^2\theta \, \frac{1}{2} W W \right] + \mathcal{L}^{(quarks)} + \Delta\mathcal{L}, \tag{1}$$

where

$$\Delta\mathcal{L} = \int d^2\theta \, \mu \, \mathrm{Tr}\, \Phi^2 \tag{2}$$

is the adjoint mass breaking the supersymmetry to $N = 1$ and

$$\mathcal{L}^{(quarks)} = \sum_i \left[\int d^4\theta \, \{ Q_i^\dagger e^V Q_i + \tilde{Q}_i e^{-V} \tilde{Q}_i^\dagger \} + \int d^2\theta \, \{ \sqrt{2} \tilde{Q}_i \Phi Q^i + m_i \tilde{Q}_i \Phi Q^i \} \right] \tag{3}$$

describes the n_f flavors of hypermultiplets ("quarks"), and $\tau_{cl} \equiv \theta_0/\pi + 8\pi i/g_0^2$ is the bare θ parameter and coupling constant. The $N = 1$ chiral and gauge superfields $\Phi = \phi + \sqrt{2}\, \theta \, \psi + \ldots$, and

[1] This "derivation," however, was incomplete as it did not produce all components of the "meson" superfield. Moreover, the effective low-energy theory was perturbed by a relevant operator (the mass term for the mesons) and did not flow to the Seiberg's magnetic theory correctly. We thank P. Argyres for discussions on this point.

$W_\alpha = -i\lambda + \frac{i}{2}\,(\sigma^\mu\,\bar\sigma^\nu)_\alpha^\beta\,F_{\mu\nu}\,\theta_\beta + \ldots$ are both in the adjoint representation of the gauge group, while the quarks are taken in the fundamental representation. In the limit $m_i \to 0$, and $\mu \to 0$, these models possess an exact flavor symmetry, $U(n_f) \times Z_{2n_c - n_f}$ or $SO(2n_f) \times Z_{2n_c+2-n_f}$, for $SU(n_c)$ or $USp(2n_c)$ gauge groups, respectively. In the equal quark mass limit, the symmetry of the symplectic gauge theory is reduced to $U(n_f)$. The models are asymptotically free as long as $n_f < 2n_c$ (for $SU(n_c)$ gauge theory) or $n_f < 2n_c + 2$ (for $USp(2n_c)$).

We find

$$\mathcal{N} = \sum_{r=0}^{\min\{n_f, n_c-1\}} (n_c - r)\, {}_{n_f}C_r \tag{4}$$

semi-classical solutions for $SU(n_c)$ gauge theory with n_f flavors, while for $Usp(2n_c)$ theory with n_f flavors, the number of $N = 1$ vacua is

$$\mathcal{N} = \sum_{r=0}^{\min\{n_c,\, n_f\}} (n_c - r + 1)\, {}_{n_f}C_r\,. \tag{5}$$

The factor $n_c - r$ or $n_c - r + 1$ appearing in the sum originates from Witten's index for unbroken gauge group. For small number of flavors, these expressions simplify somewhat:

$$\mathcal{N}_1 = (2\,n_c - n_f)\,2^{n_f - 1}, \qquad (SU(n_c) \text{ with } n_f \le n_c); \tag{6}$$

$$\mathcal{N}_1 = (2\,n_c + 2 - n_f)\,2^{n_f - 1}, \qquad (USp(2n_c) \text{ with } \; n_f \le n_c + 1). \tag{7}$$

It is amusing that these different expressions all reproduce correctly the number of $N = 1$ vacua in the case of $SU(2)$ theory (which is a special case, both of $SU(n_c)$ and of $USp(2n_c)$) with $n_f = 0 \sim 4$,

$$\mathcal{N} = n_f + 2. \tag{8}$$

3. We next determine the possible patterns of dynamical flavor symmetry breaking in these theories. This is done most easily by studying these theories at large fixed $\mu \gg \Lambda$, $m_i \to 0$.[2] Such an analysis is possible since at large adjoint mass the low-energy effective superpotential can be read off from the bare Lagrangian by integrating out the heavy adjoint field and by adding to it the known exact instanton–induced superpotentials of the corresponding $N = 1$ theories. By minimizing the superpotential, we found in all cases the correct number of vacua Eqs.(4)-(7). $N = 1$ supersymmetry kept intact throughout guarantees that there are no phase transitions as μ is varied; we can thus determine the symmetry breaking pattern in each $N = 1$ vacuum from the first principles. The analysis is straightforward, but is not entirely trivial for large n_f because the non-perturbative effects among the low-energy degrees of freedom (dual quarks and mesons) have to be correctly taken into account despite the fact that they are in a "free magnetic phase"[5].

For instance, for $SU(n_c)$ theory with $n_f < n_c$ the effective superpotential reads

$$W = -\frac{1}{2\mu}\left[\mathrm{Tr}M^2 - \frac{1}{n_c}(\mathrm{Tr}M)^2\right] + \mathrm{Tr}(mM) + \frac{\Lambda_1^{(3n_c - n_f)/(n_c - n_f)}}{(\det M)^{1/(n_c - n_f)}}\,, \tag{9}$$

[2]We also investigated the limit $\mu \to \infty$ while $m_i \ll \Lambda$ fixed, which is suited for studying the decoupling of the adjoint fields. We checked this way the consistency with the known results about $N = 1$ theories.

where $M_i^j \equiv \tilde{Q}_i^a Q_a^j$, and $\Lambda_1 = (\mu^{n_c} \Lambda^{2n_c - n_f})^{\frac{1}{3n_c - n_f}}$ is the scale of the $N = 1$ theory. The minima of the potential are characterized by the set of vacuum expectation values (in the $m_i \to 0$ limit),

$$M = \mathrm{diag}\,(\lambda_1, \lambda_2, \ldots, \lambda_{n_f}), \tag{10}$$

$$\lambda_1 = \ldots = \lambda_r = -(n_c + r - n_f)Z, \qquad \lambda_{r+1} = \ldots = \lambda_{n_f} = (n_c - r)Z, \tag{11}$$

where

$$Z = C\,\left(\mu^{n_c - n_f} \Lambda_1^{3n_c - n_f}\right)^{1/(2n_c - n_f)} \omega^k, \qquad (k = 1, 2, \ldots 2n_c - n_f;\, \omega = e^{2\pi i/(2n_c - n_f)}), \tag{12}$$

with $C \sim O(1)$ a constant that depends on n_f, n_c and r. In a vacuum characterized by r, the flavor symmetry of the model is broken spontaneously as

$$U(n_f) \to U(r) \times U(n_f - r). \tag{13}$$

To avoid double counting, we can restrict $r \leq [n_f/2]$ with all k, and for the special case of $r = n_f/2$ (possible only when n_f is even), $k = 1, \cdots, n_c - n_f/2$, for each choice of r flavors out of n_f. We find a total

$$\mathcal{N}_1 = (2n_c - n_f) \cdot 2^{n_f - 1} \tag{14}$$

of such vacua, after summation over r. The number for $n_f = n_c$ is given by the same formula using the "quantum modified constraint" among the mesons and baryons following Seiberg. For $n_f \leq n_c$ the above exhausts the number of the vacua. In the case $n_f = n_c + 1$ we used the appropriate effective Lagrangian involving mesons and baryons to find that there are $\mathcal{N}_1$ vacua with various symmetry breaking (13) plus one vacuum with no flavor symmetry breaking. The total number $\mathcal{N}_1 + 1$ reproduces (4) correctly.

The situation for larger numbers of flavor ($n_f > n_c + 1$) is more subtle. The effective low-energy action in these cases has the form (we set the "matching scale" to unity to simplify expressions)

$$W = \tilde{q} M q + \mathrm{Tr}(mM) - \frac{1}{2\mu}\left[\mathrm{Tr} M^2 - \frac{1}{n_c}(\mathrm{Tr} M)^2\right], \tag{15}$$

where q's are n_f flavors of dual quarks [5] in the fundamental representation of the dual gauge group $SU(\tilde{n}_c)$, where $\tilde{n}_c = n_f - n_c$. The minima of the potential following from Eq.(15) can be found straightforwardly, and gives

$$\mathcal{N}_2 = \sum_{r=0}^{\tilde{n}_c - 1} {}_{n_f}C_r\,(\tilde{n}_c - r) \tag{16}$$

vacua. The solutions have a color-flavor diagonal form for q's and $\tilde{q}$'s, with r nonzero elements, d_i, $\tilde{d}_i$, where

$$d_i = \tilde{d}_i = \left[-m_i - \frac{1}{n_c + r - n_f} \sum_{j=r+1}^{n_f} m_j\right]^{1/2}, \qquad i = 1, 2, \ldots, r. \tag{17}$$

The meson vacuum expectation value (VEV) is orthogonal to the squark VEVS,

$$M = \mathrm{diag}\,(0, 0, \ldots, 0, \lambda_{r+1}, \lambda_{r+2}, \ldots, \lambda_{n_f}), \qquad \lambda_i = \mu \left[m_i + \frac{1}{n_c + r - n_f} \sum_{j=r+1}^{n_f} m_j \right]. \tag{18}$$

All VEVS of fields carrying flavor quantum numbers thus vanish in the limit $m_i \to 0$, showing that the flavor symmetry remains unbroken in this class of vacua.

The problem is that the number of vacua found this way is too small, since we know that the exact number of vacua is $\mathcal{N}$ (Eq.(4)), $\mathcal{N} > \mathcal{N}_2$. Where are other vacua?

This apparent puzzle can be solved once the nontrivial $SU(\tilde{n}_c)$ instanton effects are taken into account properly.[3] If the meson vacum expectation values have rank n_f, the dual quarks can be integrated out, leaving the effective superpotential,

$$W_{\mathit{eff}} = -\frac{1}{2\mu} \left[\mathrm{Tr} M^2 - \frac{1}{n_c} (\mathrm{Tr} M)^2 \right] + \mathrm{Tr}(Mm) + \Lambda_1^{(3n_c - n_f)/(n_c - n_f)} (\det M)^{1/(n_f - n_c)}. \tag{19}$$

Minimization of this effective action gives $\mathcal{N}_1 = (2n_c - n_f) \cdot 2^{n_f - 1}$ solutions, having the same forms as Eq.(10)-Eq.(12). At this point, one can make a highly nontrivial consistency check: by changing $r \to n_f - r$ and rearranging terms, one shows that the total number of quantum vacua is equal to

$$\mathcal{N}_1 + \mathcal{N}_2 = \sum_{r=0}^{n_c - 1} (n_c - r)_{n_f} C_r = \mathcal{N}, \tag{20}$$

i.e., equal to the total number of semi-classical vacua.

We find therefore that there are two types of vacua: the first of them, with finite VEVS of mesons (in $m_i \to 0$ limit), are present for all values of flavors. They are classified by an integer $r \le [n_f/2]$, and the flavor symmetry is spontaneously broken as $U(n_f) \to U(r) \times U(n_f - r)$. In the second type of vacua, present only for large flavors ($n_f \ge n_f + 1$), the flavor symmetry remains unbroken. The second type of vacua are closely related to the emergence of the dual gauge group of Seiberg.

The analysis in the case of $USp(2n_c)$ models is similar, but the result is qualitatively different. We find again two types of $N = 1$ vacua. The first type of vacua has finite meson vacuum expectation values $M^{ij} \propto J^{ij}$ (symplectic matrix) with the flavor $SO(2n_f)$ symmetry broken as

$$SO(2n_f) \to U(n_f), \tag{21}$$

in *all* vacua of this class. This phenomenon is quite reminiscent of what is believed to occur in the standard QCD. The number of this type of vacua is given by

$$\mathcal{N}_1 = (2\, n_c + 2 - n_f)\, 2^{n_f - 1}, \tag{22}$$

[3]In fact, a related puzzle is how Seiberg's dual Lagrangian [5] - the first two terms of Eq. (15) - can give rise to the right number of vacua for the massive $N = 1$ SQCD with $n_f > n_c + 1$. By following the same method as below but with $\mu = \infty$, we do find the correct number (n_c) of vacua.

which is the number of vacua for ($n_f < n_c + 2$).

As in the $SU(n_c)$ case, when the number of the flavor is sufficiently large ($n_f \geq n_c + 2$) we find also another class of vacua in which the flavor $SO(2n_f)$ symmetry is unbroken. There are

$$\mathcal{N}_2 = \sum_{r=0}^{n_f - n_c - 2} (n_f - n_c - 1 - r)\, {}_{n_f}C_r \tag{23}$$

of them, and together with those of the first group, they make up the total number

$$\mathcal{N} = \mathcal{N}_1 + \mathcal{N}_2 = \sum_{r=0}^{n_c} (n_c + 1 - r)\, {}_{n_f}C_r \tag{24}$$

which is the correct number of $N = 1$ vacua for $USp(2n_c)$ (see Eq.(5)).

4. We now seek for a microscopic understanding of the mechanism of dynamical flavor symmetry breaking. We do so by studying the $N = 2$ vacua on the Coulomb branch which survive $\mu \neq 0$ perturbation. We start from the auxiliary genus $n_c - 1$ (n_c) curves for $SU(n_c)$ ($USp(2n_c)$) theories

$$y^2 = \prod_{k=1}^{n_c} (x - \phi_k)^2 + 4\Lambda^{2n_c - n_f} \prod_{j=1}^{n_f} (x + m_j), \quad SU(n_c),\ n_f \leq 2n_c - 2, \tag{25}$$

with ϕ_k subject to the constraint $\sum_{k=1}^{n_c} \phi_k = 0$, and

$$xy^2 = \left[x \prod_{a=1}^{n_c} (x - \phi_a^2)^2 + 2\Lambda^{2n_c + 2 - n_f} m_1 \cdots m_{n_f} \right]^2 - 4\Lambda^{2(2n_c + 2 - n_f)} \prod_{i=1}^{n_f} (x + m_i^2), \quad USp(2n_c). \tag{26}$$

The VEVS of a_{Di}, a_i are constructed as integrals over the non-trivial cycles of the meromorphic differentials on the curves. We require that the curve is maximally singular, i.e. $n_c - 1$ (or n_c for $USp(2n_c)$) pairs of branch points to coincide: this determines the possible values of $\{\phi_a\}$'s. These correspond to the $N = 1$ vacua, with the particular $N = 1$ perturbation, Eq.(2). Note that as we work with generic and nonvanishing quark masses, this is an unambiguous procedure to identify all the $N = 1$ vacua of our interest. [4]

We find in this way precisely the same number ($\mathcal{N}$) of $N = 1$ vacua, where $\mathcal{N}$ was determined earlier by the semi-classical and large μ analyses. In each vacuum, there are $n_c - 1$ (or n_c) different kinds of massless magnetic monopoles, corresponding to maximal Abelian subgroup of $SU(n_c)$ or of $USp(2n_c)$.

At *small* generic quark masses, we observe that these singularities group into approximate multiplets of vacua, with multiplicities ${}_{n_f}C_r$, $r = 0, 1, 2, \ldots, [n_f/2]$, in the case of $SU(n_c)$, while they appear in $2^{n_f - 1}$-plets plus certain number of other vacua, in the case of $USp(2n_c)$ theories. Their positions are compatible with the approximate discrete symmetries, $Z_{2n_c - n_f}$ or $Z_{2n_c + 2 - n_f}$. We have

[4] There are other kinds of singularities of $N = 2$ QMS at which, for instance, three of the branch points meet. These correspond to $N = 1$ vacua, selected out by different types of perturbations such as $\mathrm{Tr}\Phi^3$, which are not considered here.

made an extensive numerical study in the case of rank two gauge groups with all possible numbers of flavor, as well as general analytical study of these phenomena for higher-rank groups.

As $m_i \to 0$ (or equal mass limit in the case of $SU(n_c)$) each multiplet of vacua collapse into one multiple vacuum. This behavior might suggest a more or less straightforward generalization of what occurs in $SU(2)$ gauge theories, mentioned at the beginning. Indeed, monopoles can acquire nontrivial flavor quantum numbers as shown by Jackiw and Rebbi [12] through the fermion zero modes. In $SU(n_c)$ theories, by acting fermion zero mode operators d_i, $d_j^\dagger$ on the monopole state $|\Omega\rangle$, such as

$$d_i^\dagger|\Omega\rangle,\ d_{i_1}^\dagger d_{i_2}^\dagger|\Omega\rangle,\ \ldots\ , d_{i_1}^\dagger \cdots d_{i_{n_f}}^\dagger|\Omega\rangle, \tag{27}$$

we find semi-classical monopoles belonging to anti-symmetric tensor representations of $U(n_f)$. It might appear then the dynamical flavor symmetry breaking (13) is caused by the condensation of such monopoles. As mentioned earlier, however, this picture would lead to far too many Nambu–Goldstone multiplets (except for $r = 0, 1$). The same analysis for $USp(2n_c)$ case shows that the semi-classical monopoles are in the spinor representation of $SO(2n_f)$, and their condensation would give the symmetry breaking (21) and the number of vacua 2^{n_f-1}. We would again run into a paradox of having a too-large $SU(2^{n_f-1})$ symmetry.

Actually the theories avoid falling into this kind of paradox, but do so in a subtle way. Let us discuss below physics of $SU(n_c)$ and $USp(2n_c)$ gauge theories separately.

5. In the $SU(n_c)$ case, the $N = 1$ vacua can all be generated from the various classes of superconformal theories with $m_i = \mu = 0$, by perturbation by masses m_i. The first type of vacua (with multiplicity $\mathcal{N}_1$) correspond to the curves

$$y^2 \sim x^{2r}(x-\alpha_1)^2 \cdots (x-\alpha_{n_c-r-1})^2(x-\beta)(x-\gamma), \qquad r = 0, 1, 2, \ldots, [n_f/2], \tag{28}$$

that is

$$\mathrm{diag}\,\phi = (\underbrace{0, 0, \ldots, 0}_{r}, \phi_1, \ldots \phi_{n_c-r}), \qquad \sum_{a=1}^{n_c-r} \phi_a = 0, \tag{29}$$

with ϕ_a's chosen such that the nonzero $2(n_c - r - 1)$ branch points are paired. These correspond to the so-called class 1 ($r < n_f/2$) and 3 ($r = n_f/2$, with $n_c - n_f/2$ odd) superconformal theories [8], while the case, $r = n_f/2$, $n_c - n_f/2$ even, may be interpreted as belonging to class 4. Since these adjoint VEVS break the discrete symmetry spontaneously, they appear in $2n_c - n_f$ copies.[5] When (generic) quark masses are turned on, these vacua split into ${}_{n_f}C_r$-plet of single vacua. The second class of vacua stem from the (trivial) superconformal theory

$$y^2 \sim x^{2\tilde{n}_c}(x^{n_c-\tilde{n}_c} - \Lambda^2)^2, \qquad \tilde{n}_c = n_f - n_c, \tag{30}$$

[5]There is an exception to this. In the case of $r = n_f/2$ with n_f even, the explicit configuration of ϕ_a's can be found by using the Chebyshev polynomials. This vacuum respects Z_2 subgroup of the $Z_{2n_c-n_f}$ symmetry, showing that it appears in $n_c - n_f/2$ copies rather than $2n_c - n_f$. This fact is crucial in the vacuum counting below Eq.(38).

corresponding to the singularity

$$\mathrm{diag}\,\phi = (\underbrace{0, 0, \ldots, 0}_{\tilde{n}_c}, \Lambda\,\omega, \ldots, \Lambda\,\omega^{n_c-\tilde{n}_c}) \tag{31}$$

with $\omega = e^{2\pi i/(n_c-\tilde{n}_c)}$. Actually there is no vacuum of the first type with $r = \tilde{n}_c$.

The most detailed description of these $N = 1$ vacua comes from the considerations based on nonrenormalization theorem of the Higgs branch metric [10]. The first class of vacua with given r is an $SU(r) \times U(1)^{n_c-r}$ gauge theory with n_f "quarks" and $n_c - r$ singlet monopoles e_k's, with an effective Lagrangian,

$$W_{nonbar} = \sqrt{2}\mathrm{Tr}(q\phi\tilde{q}) + \sqrt{2}\psi_0 \mathrm{Tr}(q\tilde{q}) + \sqrt{2}\sum_{k=1}^{n_c-r-1} \psi_k e_k \tilde{e}_k + \mu \left(\Lambda \sum_{i=0}^{n_c-r-1} x_i \psi_i + \frac{1}{2}\mathrm{Tr}\phi^2 \right), \tag{32}$$

where ϕ and ψ_k's are part of the $SU(r) \times U(1)^{n_c-r}$ $N = 2$ vector multiplets and $x_i \sim O(1)$ constants. These are at the roots of the so-called "non-baryonic" branches [10], where they meet the Coulomb branch. They describe an infrared-free (i.e., non asymptotic free) effective theory for $r < n_f/2$. We now add the mass terms

$$\mathrm{Tr}(mq\tilde{q}) - \sum_{k,i} S_k^i m_i e_k \tilde{e}_k \tag{33}$$

and minimize the potential. We find ${}_{n_f}C_r$ solutions characterized by the vacuum expectation values (q and $\tilde{q}$ having color-flavor diagonal form, with nonvanishing elements, d_i and $\tilde{d}_i$)[6]

$$\psi_0 = -\frac{1}{\sqrt{2}\,r}\sum_{i=1}^{r} m_i, \qquad \psi_k = O(m_i), \tag{34}$$

$$d_i\tilde{d}_i = -\mu\left(m_i - \frac{1}{r}\sum_{j=1}^{r} m_j \right) - \frac{1}{\sqrt{2}\,r}\mu\Lambda x_0; \qquad e_k\tilde{e}_k \sim \mu\Lambda. \tag{35}$$

The multiplicity ${}_{n_f}C_r$ arises from the choice of r (out of n_f) quark masses used to construct the solution. In the massless limit we find

$$d_i = \tilde{d}_i \sim \sqrt{\mu\Lambda}, \qquad i = 1, 2, \ldots r : \tag{36}$$

this leads to the correct symmetry breaking pattern, $U(n_f) \to U(r) \times U(n_f - r)$.

For $r = n_f/2$, the theory at the singularity becomes a non-trivial superconformal theory. There is no description of this singularity in terms of weakly coupled local field theory. The monodromy around the singularity shows that the theory is indeed superconformal (we checked this explicitly

[6]Actually, Eq.(32) and Eq.(33) allow for a number of other solutions in which the vev of ψ_0 is of $O(\Lambda)$; these are the first group of $N = 1$ vacua found in [10]. Such solutions, involving fluctuations much larger than both m_i and μ, however, lie beyond the validity of the low-energy effective Lagrangian. They should therfore be regarded as an artefact of the approximation and must be discarded.

for $n_c = 3$ and $n_f = 4$). Careful perturbation of the curve by the quark masses shows that there are $(n_c - n_f/2)\, {}_{n_f}C_{n_f/2}$ vacua.[7]

The total number of the vacua of this type is ($n_f \le n_c$):

$$(2n_c - n_f) \sum_{r=0}^{(n_f-1)/2} {}_{n_f}C_r = (2n_c - n_f)\, 2^{n_f-1}, \quad (n_f = \text{odd}) \tag{37}$$

$$(2n_c - n_f) \sum_{r=0}^{n_f/2-1} {}_{n_f}C_r + \frac{2n_c - n_f}{2}\, {}_{n_f}C_{n_f/2} = (2n_c - n_f)\, 2^{n_f-1}, \quad (n_f = \text{even}), \tag{38}$$

which exhausts $\mathcal{N}$, Eq.(6). In Eq.(38) we have taken into account the fact that for even n_f, the vacua with $r = n_f/2$ do not transform under $Z_{2n_c-n_f}$ but only under $Z_{n_c-n_f/2}$. When $n_f > n_c$, we need to exclude the term $r = \tilde{n}_c = n_f - n_c$ from the sum because it gives the second type of vacua. We obtain therefore $\mathcal{N}_1 - (2n_c - n_f)\, {}_{n_f}C_{\tilde{n}_c}$ vacua.

As for the second group of vacua, Eq.(30), Eq.(31), they are an $SU(\tilde{n}_c) \times U(1)^{n_c - \tilde{n}_c}$ gauge theory with n_f "quarks" and $n_c - \tilde{n}_c$ singlet monopoles e_k's [10]. The effective low-energy Lagrangian for this theory is given by

$$W_{bar} = \sqrt{2}\mathrm{Tr}(q\phi\tilde{q}) + \frac{\sqrt{2}}{\tilde{n}_c}\mathrm{Tr}(q\tilde{q}) \left(\sum_{k=1}^{n_c-\tilde{n}_c} \psi_k \right) - \sqrt{2} \sum_{k=1}^{n_c-\tilde{n}_c} \psi_k e_k \tilde{e}_k + \mu \left(\Lambda \sum_{i=1}^{n_c-\tilde{n}_c} x_i \psi_i + \frac{1}{2}\mathrm{Tr}\phi^2 \right) . \tag{39}$$

where ϕ and ψ_k's are now in $SU(\tilde{n}_c) \times U(1)^{n_c-\tilde{n}_c}$ $N = 2$ vector multiplets. We add the mass terms

$$\mathrm{Tr}(mq) - \sum_{k,i} S_k^i m_i e_k \tilde{e}_k. \tag{40}$$

We find two types of vacua of the effective low-energy Lagrangian. The first type has $e_k = \tilde{e}_k = (\mu\Lambda x_k/\sqrt{2})^{1/2}$ for all $k = 1, \cdots n_c - \tilde{n}_c$. Minimizing the potential in this case, we find

$$\mathcal{N}_2 = \sum_{r=0}^{\tilde{n}_c-1} (\tilde{n}_c - r)\ {}_{n_f}C_r \tag{41}$$

$N = 1$ vacua, characterized by the vevs

$$\phi = \frac{1}{\sqrt{2}}\mathrm{diag}\,(-m_1, \ldots, -m_r, c, \ldots, c)\,; \qquad c = \frac{1}{\tilde{n}_c - r}\sum_{k=1}^{r} m_k \tag{42}$$

$$d_i, \tilde{d}_i \sim \sqrt{\mu m} \overset{m_i \to 0}{\longrightarrow} 0, \qquad\qquad e_k, \tilde{e}_k \sim \sqrt{\mu\Lambda}. \tag{43}$$

The unbroken $SU(\tilde{n}_c - r)$ gauge group gives $\tilde{n}_c - r$ vacua each. These vacua describe the vacua with unbroken $SU(n_f)$ symmetry, which are known to exist from the large μ analysis.

[7] Due to some reason, however, the naive application of the effective Lagrangian Eq. (32,33) gives the correct number.

The second type of vacua in Eqs. (39,40) has one of the $e_k = \tilde{e}_k = 0$ (hence $n_c - \tilde{n}_c = 2n_c - n_f$ choices) while $\partial W / \partial \psi_k = 0$ requires quarks to condense with $q = \tilde{q} \sim \sqrt{\mu \Lambda}$. Dropping $e_k = \tilde{e}_k = 0$ from the Lagrangian, it becomes the same as that of the non-baryonic root Eqs. (32,33) and gives $(2n_c - n_f)\, {}_{n_f}C_{\tilde{n}_c}$ vacua. This precisely compensates the exclusion of $r = \tilde{n}_c$ in the sum for the non-baryonic roots and the correct total number of vacua $\mathcal{N}_1 + \mathcal{N}_2$ is obtained.

We thus find that both the number and the symmetry properties of the $N = 1$ theories at small adjoint mass μ exactly match those found at large μ, without encountering any paradoxical situation. We postpone a discussion of physical aspects of $SU(n_c)$ theories to the end (point 7 below).

6. In the $USp(2n_c)$ gauge theories, the first type of vacua can be identified more easily by first considering the equal but novanishing quark masses. The adjoint vevs in the curve Eq.(26) can be chosen so as to factor out the behavior

$$y^2 = (x + m^2)^{2r} \, [\ldots], \quad r = 1, 2, \ldots \tag{44}$$

which describes an $SU(r) \times U(1)$ gauge theory with n_f quarks. These (trivial) superconformal theories belong in fact to the same universality classes as in the $SU(n_c)$ gauge theory as pointed out by [8]. They are therefore described by exactly the same Lagrangian Eq.(32). At each vacuum with r, the symmetry (of equal mass theory, $U(n_f)$) is broken spontaneously as

$$U(n_f) \to U(r) \times U(n_f - r) : \tag{45}$$

as in Eq.(13). When a small mass splitting is added among m_i's, each of the r vacuum split into ${}_{n_f}C_r$ vacua, leading to the total of

$$(2n_c + 2 - n_f) \sum_{r=0}^{(n_f-1)/2} {}_{n_f}C_r = (2n_c + 2 - n_f)\, 2^{n_f - 1}, \quad (n_f = \text{odd}) \tag{46}$$

$$(2n_c + 2 - n_f) \sum_{r=0}^{n_f/2-1} {}_{n_f}C_r + \frac{2n_c + 2 - n_f}{2}\, {}_{n_f}C_{n_f/2} = (2n_c + 2 - n_f)\, 2^{n_f - 1}, \quad (n_f = \text{even}), \tag{47}$$

vacua of this type, consistently with Eq. (7).[8]

In the massless limit the underlying theories possess a larger, flavor $SO(2n_f)$ symmetry. We know also from the large μ analysis that in the first group of vacua (with finite vevs), this symmetry is broken spontaneously to $U(n_f)$ symmetry always. How can such a result be consistent with Eq.(45) of equal (but nonvanishing) mass theory?

What happens is that in the massless limit various $N = 1$ vacua with different symmetry properties Eq.(45) (plus eventually other singularities) coalesce. The location of this singularity can be obtained exactly in terms of Chebyshev polynomials. At the singularity there are mutually non-local dyons and hence the theory is at a non-trivial infrared fixed point (in the example of $USp(4)$ theory with $n_f = 4$, we have explicitly verified this by determining the singularities and branch points at

[8] These $N = 1$ vacua seem to have been overlooked in [11].

finite equal mass m and by studying the limit $m \to 0$.) There is no description in terms of a weakly coupled local field theory, just as in the case $r = n_f/2$ for $SU(n_c)$ theories. Since the global flavor symmetry is $SO(2n_f)$ these superconformal theories belong to different universality classes as compared to those at finite mass. We find this behavior resonable because the semi-classical monopoles are in the spinor representation of the $SO(2n_f)$ flavor group and, in contrast to the situation in $SU(n_c)$ theories, cannot "break up" into quarks in the vector representation. They are therefore likely to persist at the singularity and makes the theory superconformal. Once the quark masses are turned on, however, the flavor group reduces to (at least) $U(n_f)$ and it becomes possible for monopoles to break up into quarks; this explains the behavior in the equal mass case.

As for the second group of vacua, the situation is more analogous to the case of $SU(n_c)$ theories. The superpotential reads in this case (by adding mass terms to Eq.(5.10) of [11]):

$$\begin{aligned} W &= \mu \left(\mathrm{Tr}\, \phi^2 + \Lambda \sum_{a=1}^{2n_c+2-n_f} x_a \psi_a \right) + \frac{1}{\sqrt{2}}\, q_a^i \phi_b^a q_c^i\, J^{bc} + \frac{m_{ij}}{2} q_a^i q_b^j\, J^{ab} \\ &+ \sum_{a=1}^{2n_c+2-n_f} \left(\psi_a\, e_a\, \tilde{e}_a + S_a^i m_i\, e_a\, \tilde{e}_a \right) , \end{aligned} \tag{48}$$

where $J = i\sigma_2 \otimes \mathbf{1}_{n_c}$ and

$$m = -i\sigma_2 \otimes \mathrm{diag}\,(m_1, m_2, \ldots, m_{n_f})\,. \tag{49}$$

By minimizing the potential, we find

$$\mathcal{N}_2 = \sum_{r=0}^{\tilde{n}_c} (\tilde{n}_c - r + 1)_{n_f} C_r \tag{50}$$

vacua, which precisely matches the number of the vacua of the second group, with squark vevvs behaving as

$$q_i, \tilde{q}_i \sim \sqrt{\mu m_i} \overset{m_i \to 0}{\longrightarrow} 0. \tag{51}$$

These are the desired $SO(2n_f)$ symmetric vacua.

7. To summarize, we have studied the dynamics of $N = 1$ $SU(n_c)$ and $USp(2n_c)$ gauge theories obtained by perturbing $N = 2$ theories with n_f hypermultiplets in the fundamental representation with a finite adjoint mass $\mu \mathrm{Tr}\Phi^2$, determining the possible flavor symmetry breaking patterns. There are vacua in confinement phase with symmetry breaking $U(n_f) \to U(r) \times U(n_f - r)$ ($r = 0, 1, \cdots, [n_f/2]$) and $SO(2n_f) \to U(n_f)$, respectively. There also are non-confining vacua with no flavor symmetry breaking for $n_f \geq n_c + 1$, $n_f \geq n_c + 2$ for $SU(n_c)$ and $USp(2n_c)$ theories, respectively.

With small but generic quark masses, the order parameter of confining vacua is indeed the condensation of magnetic monopoles for every $U(1)$ factor on the Coulomb branch à la 't Hooft, in both types of gauge theories. The massless limit, however, is non-trivial and much more interesting.

In $SU(n_c)$ theories, in vacua with $r = 1$ magnetic monopoles are in the fundamental representation of $U(n_f)$ flavor group, and are charged under one of the $U(1)$'s: flavor-singlet monopoles are charged under other $U(1)$'s. Their condensation realizes the confinement and the flavor symmetry breaking at the same time.

In vacua labelled by r, $2 \leq r < n_f/2$ but $r \neq n_f - n_c$, the grouping of the associated singularities on the Coulomb branch might suggest the condensation of monopoles in the rank-r anti-symmetric tensor representation. Actually, this does not occur. The correctness of the effective action Eq.(32) shows that the low-energy degrees of freedom of these theories are (magnetic) quarks plus a number of singlet monopoles of an effective $SU(r) \times U(1)^{n_c - r}$ gauge theory. Monopoles in a higher representation of $SU(n_f)$ flavor group probably exist semi-classically as seen in a Jackiw–Rebbi type analysis [12]. Such monopoles can be interpreted as "baryons" made of the magnetic quarks, which, interactions being infrared-free, break up before they become massless at singularities on the Coulomb branch. The condensation of the magnetic quarks induces the confinement and flavor symmetry breaking, $U(n_f) \to U(r) \times U(n_f - r)$, at the same time. This is how the system avoids falling into a paradox of having too many Nambu-Goldstone multiplets.

In the special cases with $r = n_f/2$, the interactions among the monopoles are so strong that the low-energy theory describing them is a nontrivial superconformal theory (conformal invariance explicitly broken by the adjoint or quark masses). Although the symmetry breaking pattern is known ($U(n_f) \to U(n_f/2) \times U(n_f/2)$), the low energy degrees of freedom are fields whose interactions are not described by a local action.

Finally, in the group of vacua labelled by $r = n_f - n_c$, the interactions among monopoles are described by an effective infrared-free $SU(n_f - n_c)$ gauge theory. There are two physically distinct groups of vacua in this case: one in which the magnetic quarks condense (i.e. confinement phase) with the unbroken symmetry $U(n_f - n_c) \times U(n_c)$, and the other with no magnetic-quark condensation and hence with unbroken $U(n_f)$ symmetry (i.e. the free magnetic phase).

In $USp(2n_c)$ theories, physics for non-vanishing and equal quark masses resembles that in the vacua with generic r of $SU(n_c)$ theory. In the massless limit, however, where the flavor group enlarges to $SO(2n_f)$, the situation is more similar to the $r = n_f/2$ case of $SU(n_c)$: the low-energy degrees of freedom are fields with relatively non-local interactions, and the effective theory is a non-trivial superconformal one. Although no local effective Lagrangian is available, we know that the flavor $SO(2n_f)$ symmetry is spontaneously broken to diagonal $U(n_f)$ symmetry in all confining vacua. For large number of flavor, there are also vacua in free-magnetic phase.

A more extensive account of our analysis will appear elesewhere.

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